



A Sub-Nyquist Radar Demo System: Hardware and Software

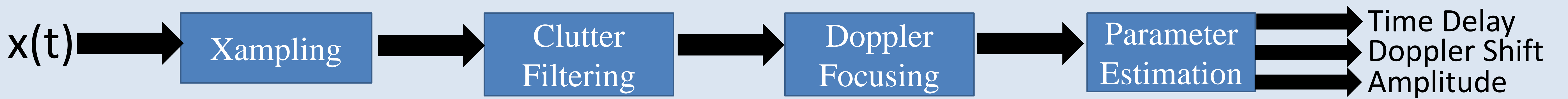
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Motivation and Goals

- High resolution radar requires high bandwidth signals
- Wideband signals need a complex analog front end receiver design which consumes high power
- Digital processing of wideband signals requires large memory and large computational power
- We present a sub-Nyquist sampling and recovery method implemented in hardware which reduces the rate by 30 fold
- This approach provides both simple recovery and robustness to noise by performing beamforming on the low rate samples
- Clutter rejection is also performed on the sub-Nyquist samples by adapting standard methods to our setting

Sub-Nyquist Radar Algorithm



- Xampling**– A process of sampling a signal at a low rate in such a way that preserves the required information
- Clutter Filtering** – Adaptation of standard clutter algorithms to fit our low rate samples
- Doppler Focusing** - A method of digitally beamforming the low rate samples which is both numerically efficient and robust to noise
- Estimation** – A modified OMP, matched to our samples, produces target locations and Doppler frequencies

Input Signal Model

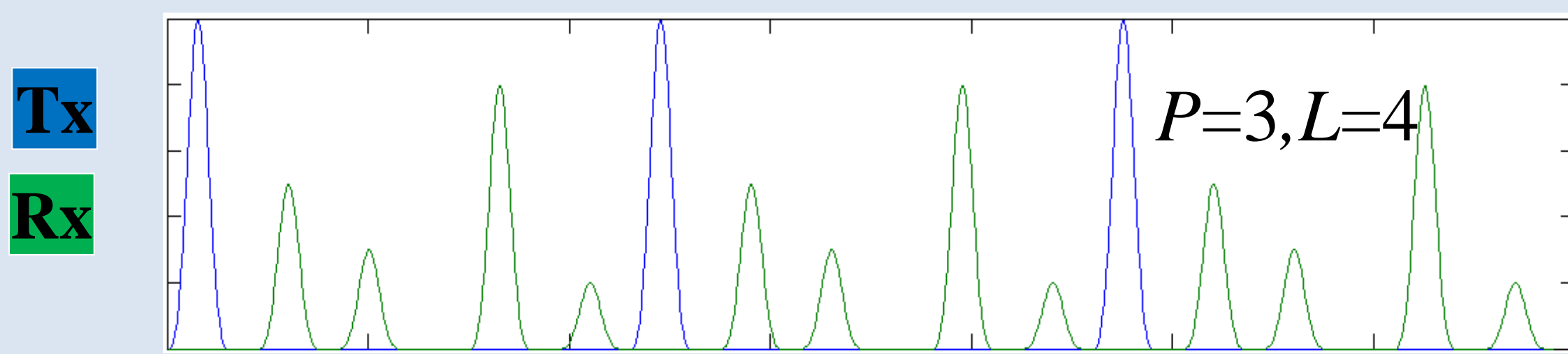
- L targets, each defined by 3 degrees of freedom: amplitude α_ℓ , delay τ_ℓ , and Doppler frequency ν_ℓ

- After transmitting P equispaced high-bandwidth pulses $h(t)$, the received signal*:

$$x(t) = \sum_{p=0}^{P-1} \sum_{l=0}^{L-1} \alpha_l h(t - \tau_l - p\tau) e^{-j\nu_l p\tau}$$

(* some assumptions on target dynamics are needed for this model)

- This is an FRI model as $x(t)$ is completely defined by $3L$ parameters



- The signal's Fourier coefficients contain the required parameters:

$$c_p[k] = \frac{1}{\tau} \int_0^\tau x_p(t) e^{-j2\pi kt/\tau} dt = \frac{1}{\tau} H(2\pi k/\tau) \sum_{l=0}^{L-1} \alpha_l e^{-j2\pi k\tau_\ell/\tau} e^{-j\nu_l p\tau}$$

- Standard radar methods sample and process at the Nyquist rate

Doppler Focusing

- Transforms a simultaneous Delay-Doppler estimation problem into a set of delay-only problems with specific Doppler frequency

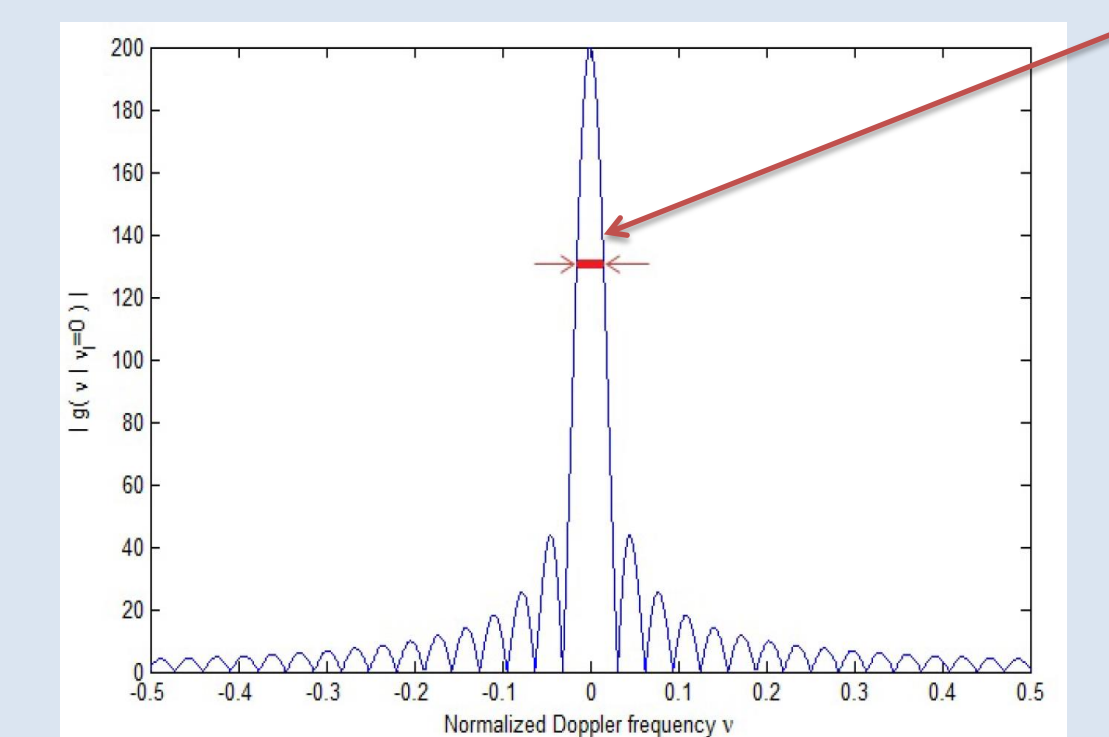
- Focusing on Doppler frequency ν for sampled Fourier coefficients:

$$\Psi_\nu[k] = \sum_{p=0}^{P-1} \tilde{c}_p[k] e^{j\nu p\tau} = \frac{1}{\tau} H(2\pi k/\tau) \sum_{l=0}^{L-1} \alpha_l e^{-j2\pi k\tau_\ell/\tau} \sum_{p=0}^{P-1} e^{j(\nu - \nu_\ell)p\tau}$$

Spectral analysis problem Focusing term

- Focusing term

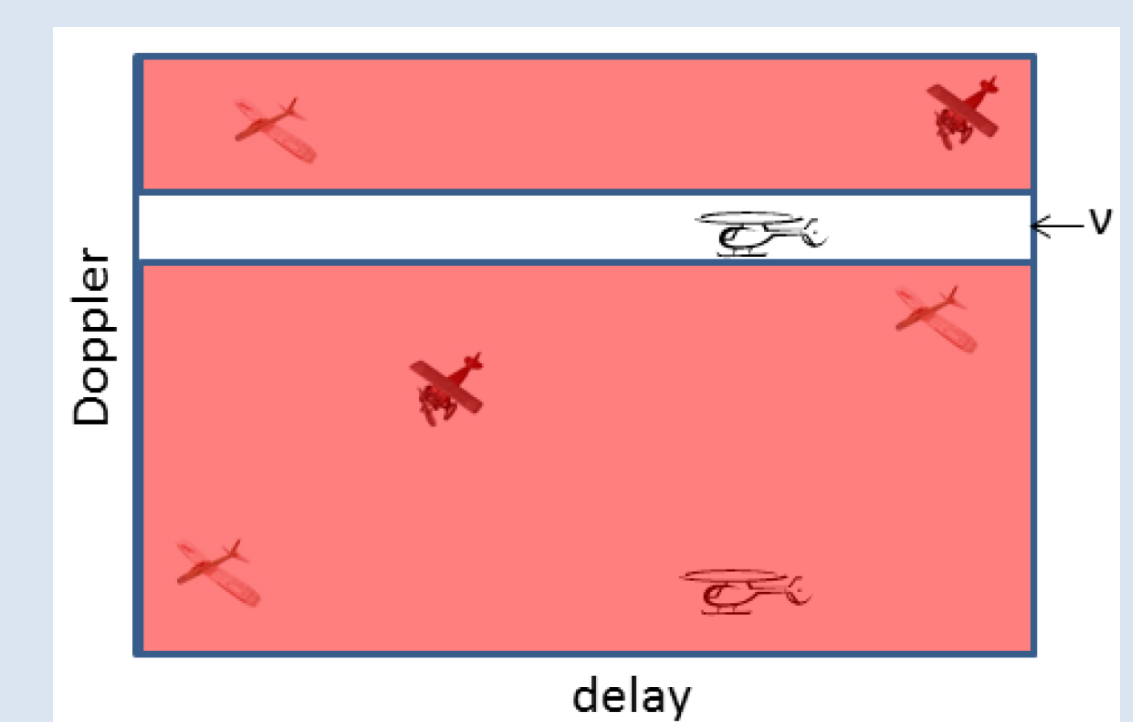
$$\sum_{p=0}^{P-1} e^{j(\nu - \nu_\ell)p\tau} \cong \begin{cases} P, & |\nu - \nu_\ell| < \pi/P\tau \\ 0, & \text{otherwise} \end{cases}$$



- Coherent integration of echoes from different pulses creates a single superimposed pulse. SNR scaling is linear with P

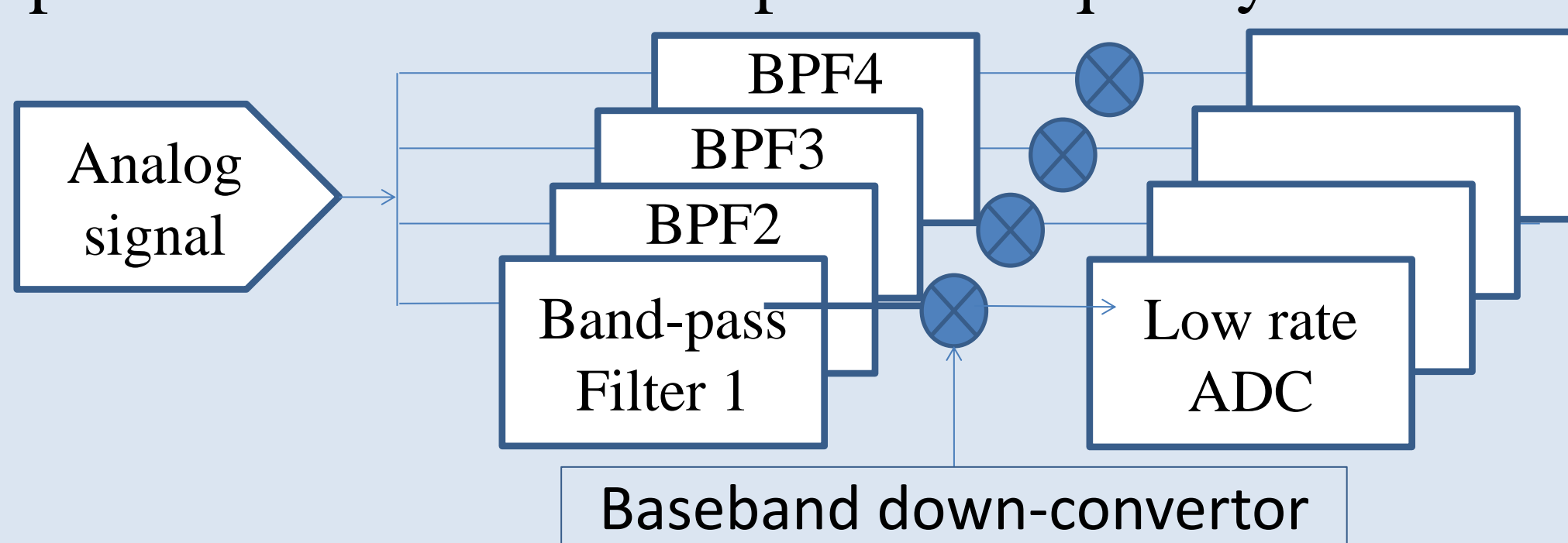
- Instead of detecting targets in the delay- Doppler plane, Doppler focusing creates slices in which targets are detected using delay only

- A hard 2D estimation problem is efficiently reduced into several easier 1D problems.**



Xampling Scheme – Acquiring Fourier Coefficients

- The signal's parameters are embodied in its Fourier coefficients
- Multichannel analog processing and low rate sampling scheme are used to extract spectral information for specific frequency bands:



- Calculating Fourier coefficients is performed digitally after sampling

Clutter Filtering

- The target signal is contaminated with clutter + thermal noise:
 $y(t) = x(t) + c(t) + n(t)$
- Assume the clutter interference is modelled as “colored” noise - a WSS random process whose spectrum is Gaussian:

$$S_c(f) = P_c \cdot \frac{1}{\sqrt{2\pi}\sigma_c} \cdot \exp\left[-\frac{(f - f_c)^2}{2\sigma_c^2}\right]$$

- Clutter + Thermal Noise autocorrelation matrix:

$$M(m, n) = (P_c/P_N) e^{-2(\pi\sigma_c T \cdot (m-n))^2} e^{-j \cdot 2\pi(m-n)f_c} + \delta_{m,n}$$

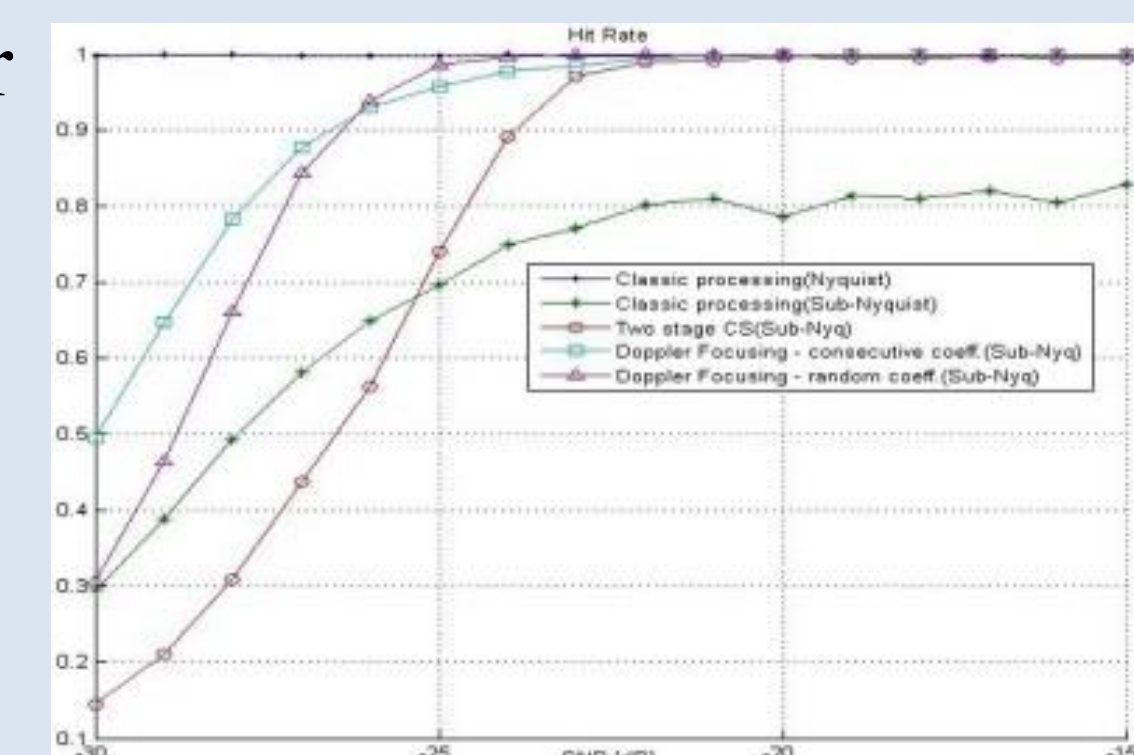
- Filtering is performed by using the whitening matrix $M^{-\frac{1}{2}}$ to whiten the interference and proceeding with Doppler focusing.



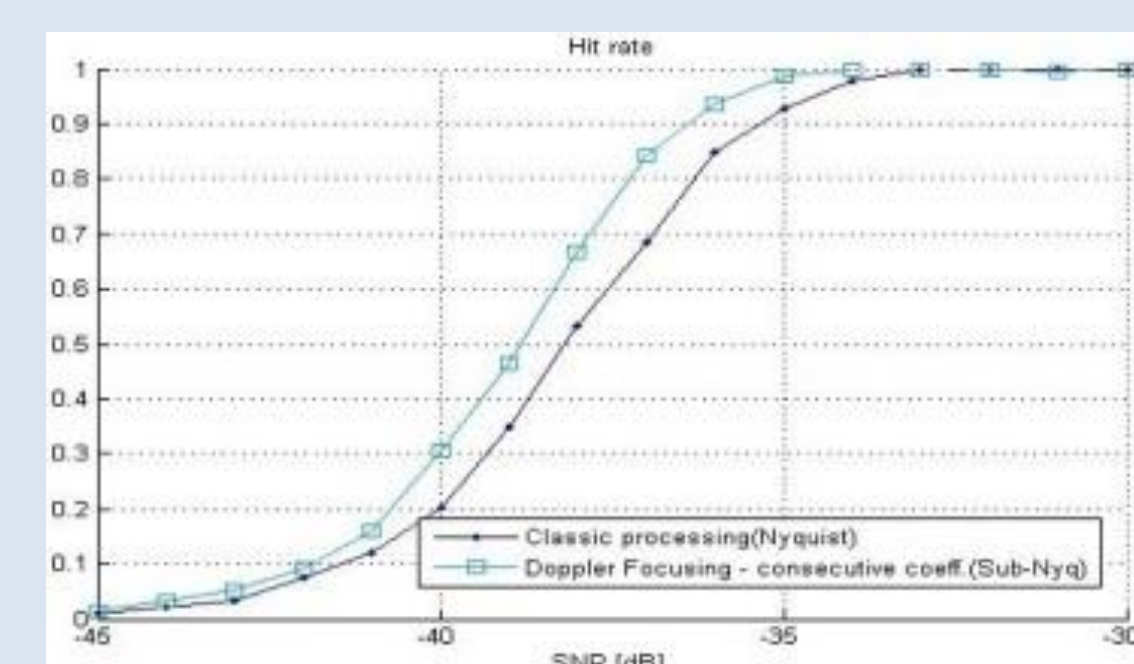
Simulation Results

- Measure performance by “hits” and RMS error

- A “hit” is a Delay-Doppler estimate in the interior of an ellipse around the true target **one tenth the Nyquist Rate** and at **-25dB SNR**, Doppler focusing achieves performance **equivalent to matched filter** processing sampling at the **Nyquist rate**

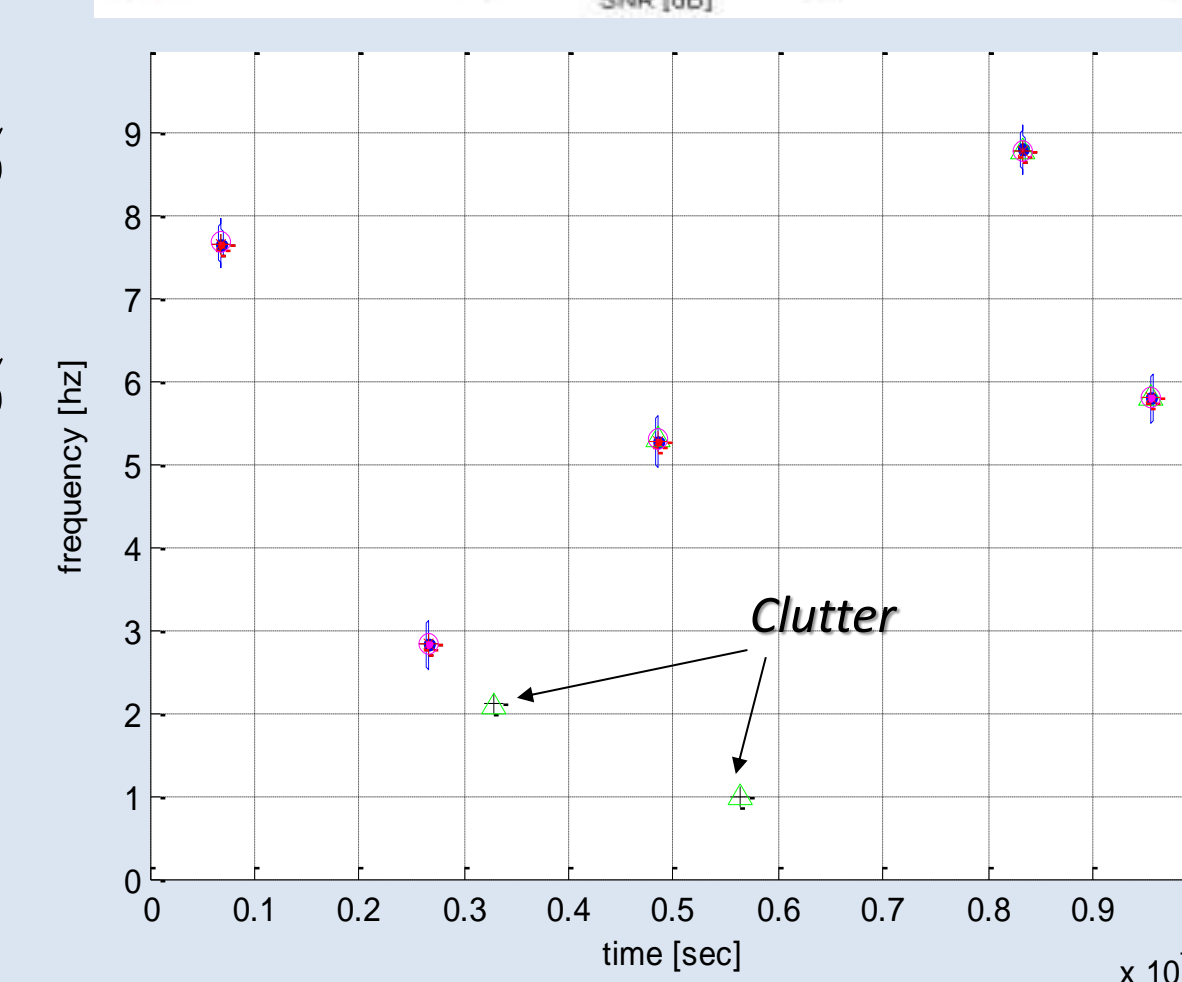


- When we concentrate the signal's energy contents in the sampled frequencies, Doppler focusing outperforms matched filtering at Nyquist rate



- Under SNR of -16dB and 100 pulses used:
- Without clutter filtering, only 3 out of 5 targets are detected
- Using clutter filtering algorithm, all 5 targets are detected.

- The performance of sub-Nyquist algorithm is equivalent to classic Nyquist rate processing.**



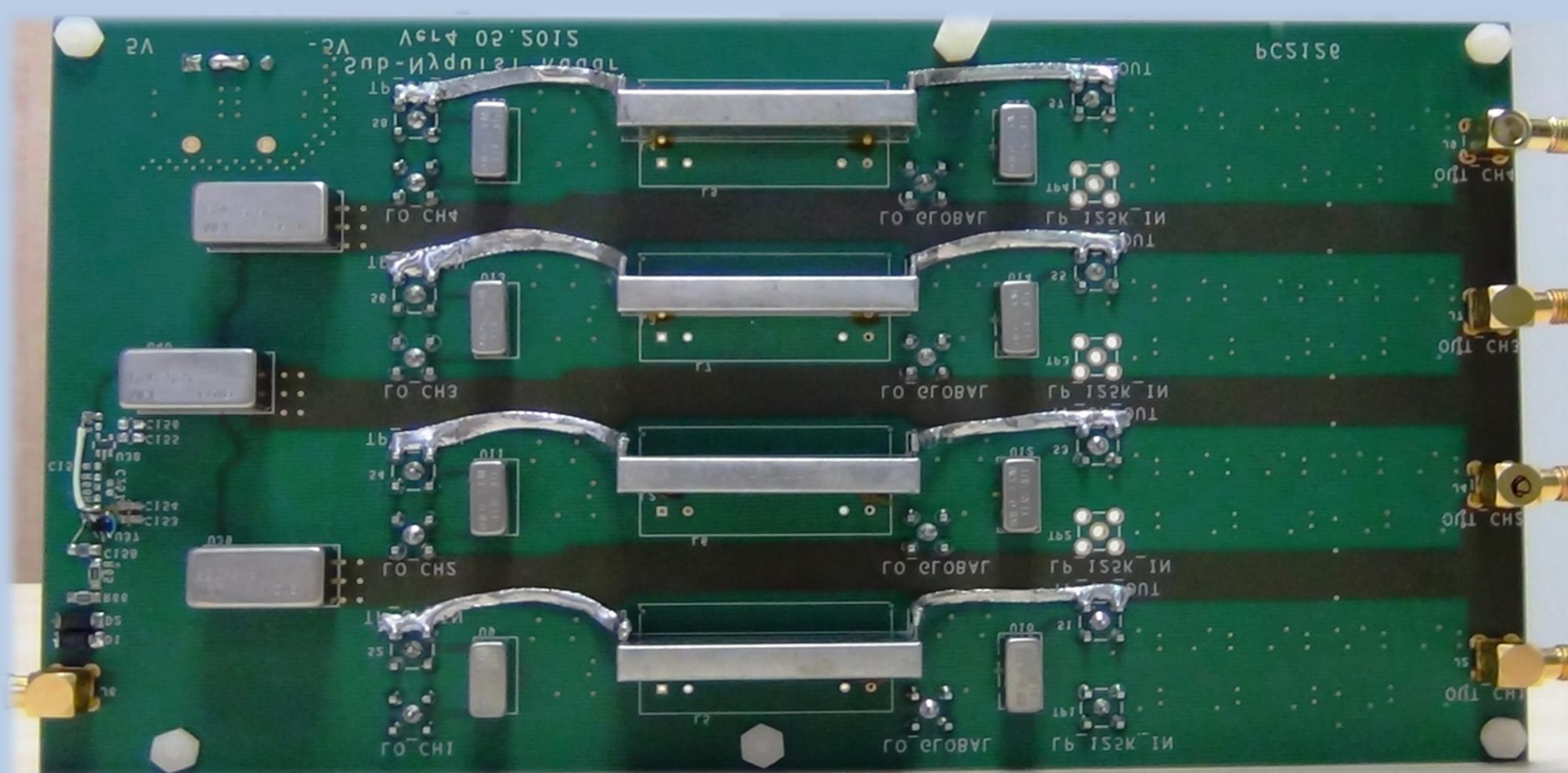
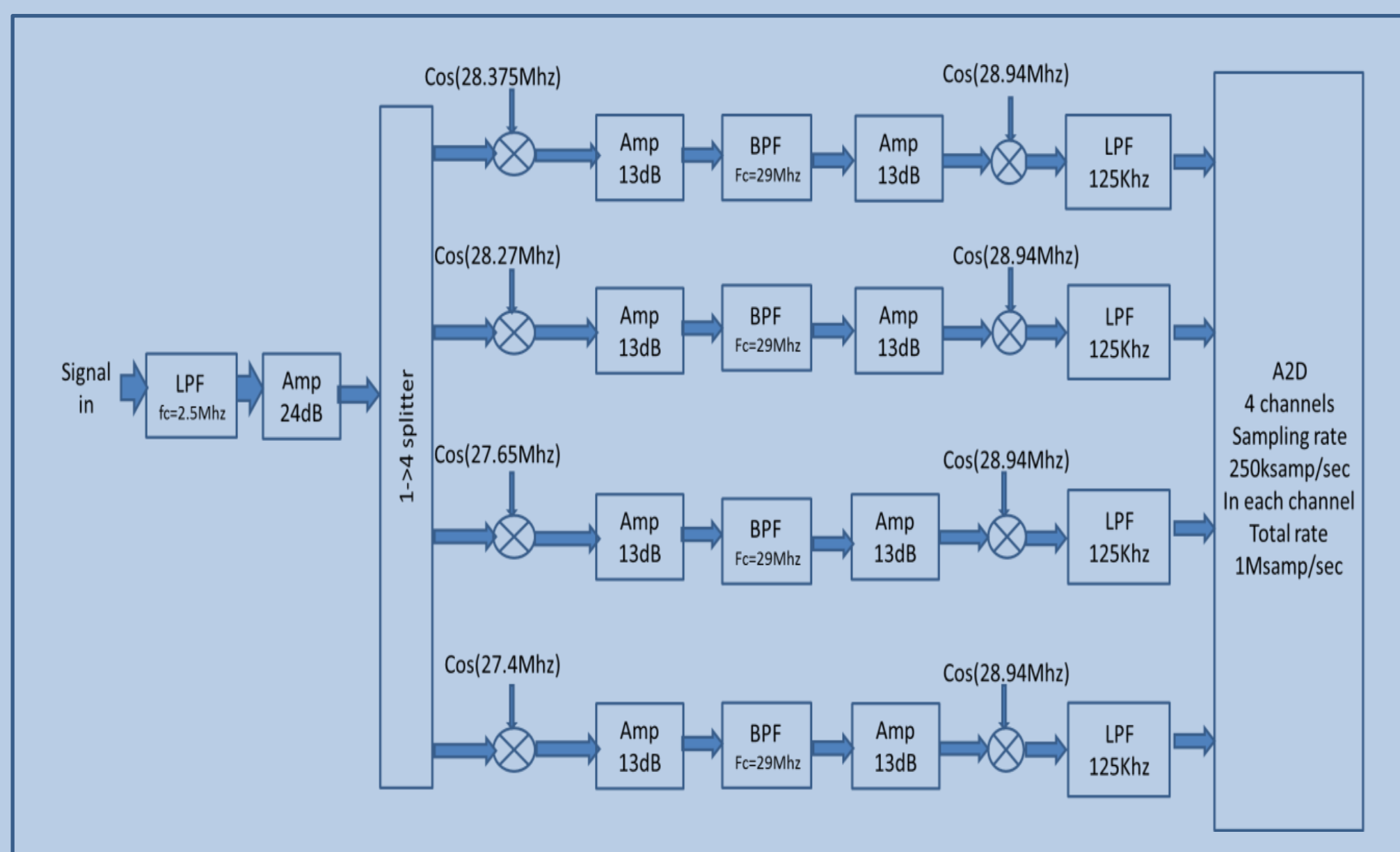
Sub-Nyquist Radar Sensing

Hardware and Supporting System

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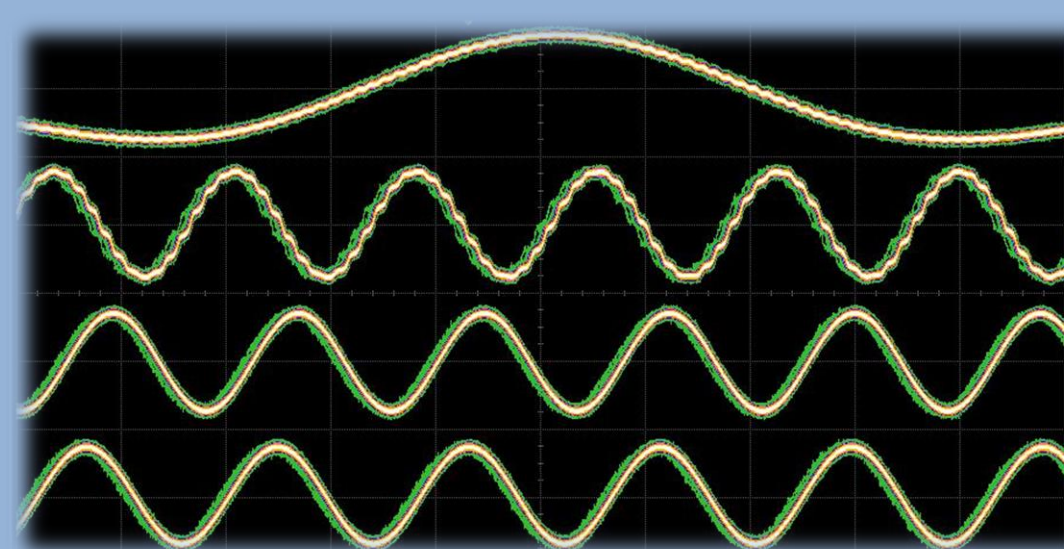
Pulse Analog Xampler



- Input signal BW < 150MHz
- Crystal filter BW 70KHz
- Modular and flexible design
- Dynamic range 65dB

Supporting Hardware – NI System

3 NI Flex Rio 7965R FPGA and NI 5781 Baseband transceiver create 5 local oscillators waveforms with constant starting phase



NI 6672 timing and synchronization module distribute clock and trigger signals

NI 6123 4 channels simultaneous A/D @ 250Ksamp/sec per channel

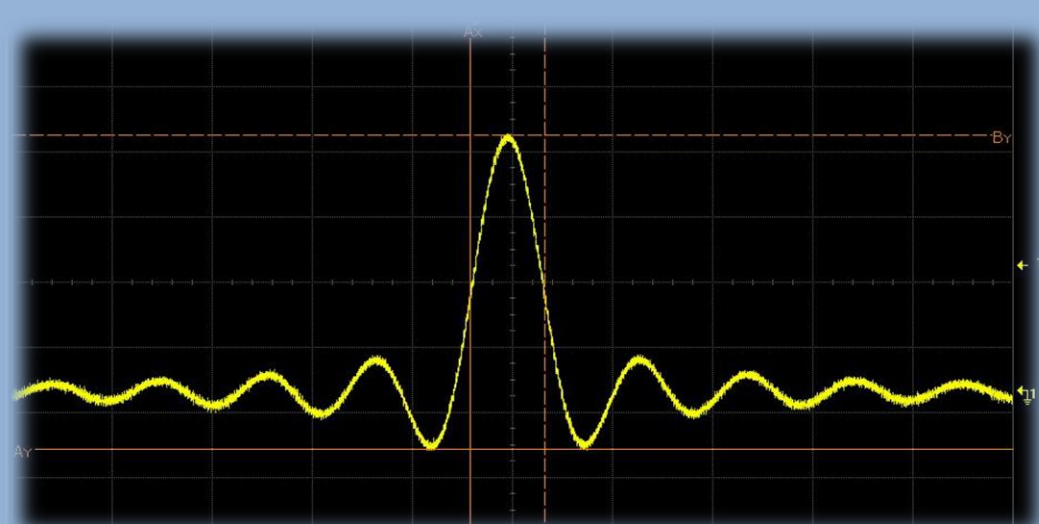
NI 4130 Power supply to Pulse Xampler



System Challenges:

- Start all devices at the same time with skew less than 1nsec
- Good synchronization- Low clock jitter and small clock drifts between devices
- Connectivity- AWR RF simulation environment to LabView

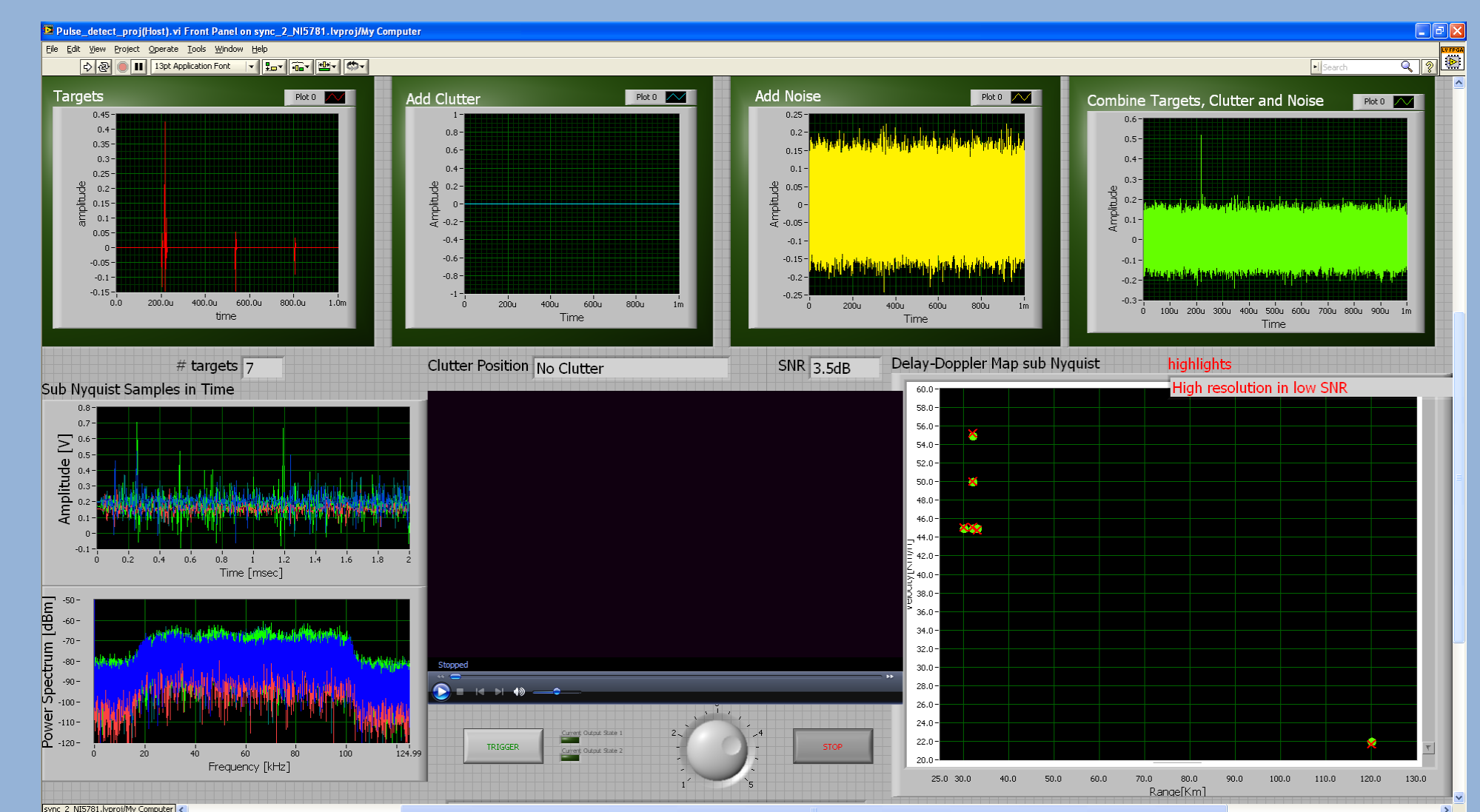
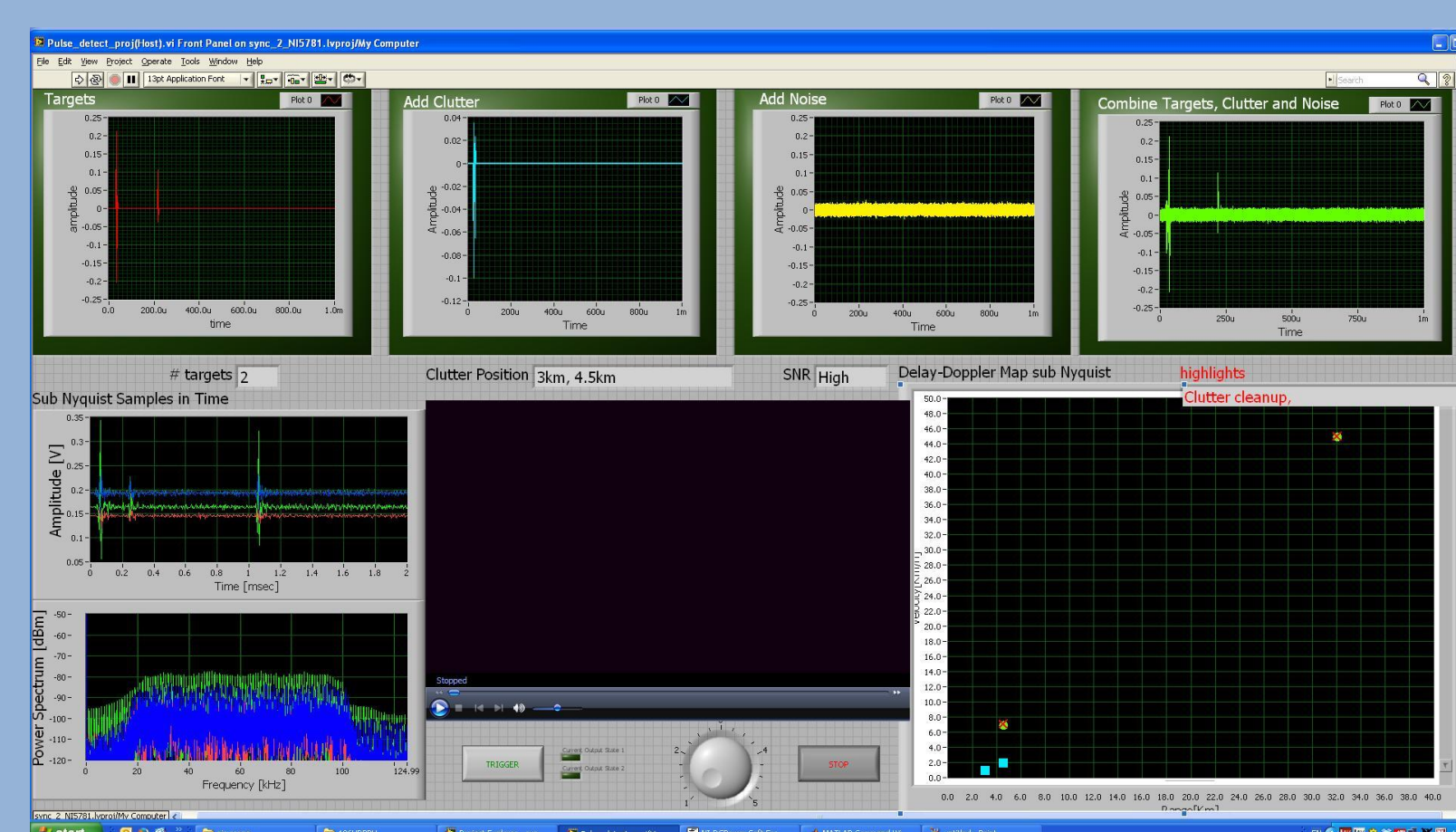
NI 8133 I7 controller
Run AWR , LabView and MATLAB script



NI 5451 Arbitrary Waveform Generator

NI 5690 RF amplifier

LabView based GUI Software



Measurements Results

RF signal – 10 MHz width
Average SNR=0dB include
2 clutter targets

4 channels sampled at 250 kHz each

Delay-Doppler Map



Xampling

DFT for each channel

Estimation Algorithm

